

**A NUMERICAL APPROACH FOR JOULIN'S MODEL  
OF A POINT SOURCE INITIATED FLAME**

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**Abstract**

For the simulation of a spherical flame initiated by a point source, Joulin has proposed a model using a fractional differential equation of order  $1/2$  whose right-hand side has got a singularity. It is known that the exact solution of this equation undergoes a bifurcation with respect to an energy input parameter. In this paper we propose to use fractional versions of various backward differentiation formulas to obtain a numerical solution of the equation. It turns out that the algorithms under consideration are able to handle the problem in a very satisfactory way. In particular we find a useful technique for the determination of the critical value of the bifurcation parameter.

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*Key Words and Phrases:* Caputo derivative, bifurcation, singular fractional differential equation, backward Euler method, flame propagation

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